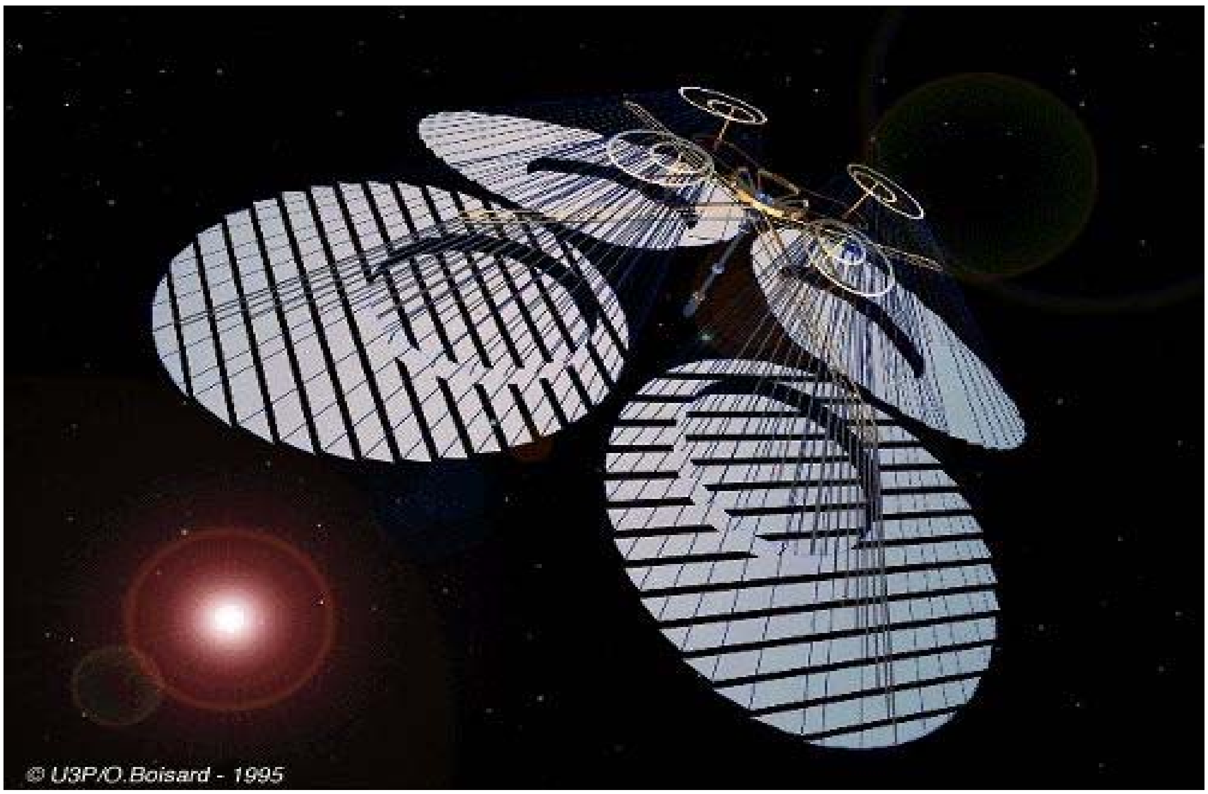
Sailing To Mars



An example of an educational document on Physics and Astronomy for advanced science education on high school level or early university. Written by Torvald Hessel, all rights reserved, Friends of the Austin Planetarium February 2005.

Written and designed by Torvald Hessel

You might have noticed that Mars is lately often in the news. The two Mars rovers Spirit and Opportunity are roving around on the surface of the red planet for more then a year now and have made many wonderful discoveries and observations. Of course the things we read in the media are concerning the *results* of the mission and the discoveries made. Naturally there are many ore aspects of a mission to Mars; after all we flew from Earth to a completely different planet! In case of the Mars rovers the trip took about 10 months, in which the craft traveled more than 800 *million* kilometers! The average speed was therefore more then 30 kilometers per second! That is very fast compared to the speed of say a car or a plane, but it still took 10 months to get to Mars. Let's take a closer look to this example. The distance between the Earth and Mars are at closest approach 7.83×10^7 km. Let us assume we would like to leave for Mars today, and we have equipped our spaceship with an engine that gives us a continuous acceleration of 5 m/s^2.

Problem 1:

a) Calculate how many days the trip would take
b) What will the speed be when we arrive at Mars?
c) This 'rather high' speed is of course a problem because we cannot just step out onto the surface of Mars. To solve this problem we can do the following: we accelerate to half the distance to mars; then we turn our spaceship around and decelerate again with 5 m/s^2. Calculate again how long our trip will take.

As you see a trip to Mars does not have to take long at all! You might wonder why then it takes NASA ten months to get to Mars. The reason is quite simple: imagine our spacecraft weights 10 metric tons, and we perform our trip like we calculated in problem 1C.

Problem 2:

a) Calculate the needed energy for the trip
b) If we assume our engine uses hydrogen as fuel, how much kg hydrogen is then needed?
c) Of course oxygen is needed to burn hydrogen. What is then the total weight of fuel we have to take along?

Of course it is clear that this is quite impossible. Firstly it would take forever to produce such an amount of hydrogen and oxygen, but a bigger problem is that the hydrogen and oxygen will have to be launched into space as well. Also our simple calculation above ignored the fact that we also have to accelerate the fuel itself. As long as we have not discovered a new fuel or propulsion system that would require a major cut in fuel use, traveling through space at those speeds will remain to be science fiction.

Instead NASA is using different techniques to speed up a spacecraft at minimal cost and fuel use. One technique is called a *gravity-sling*. Take a look at *figure 1*, the long curvy line indicates the trajectory of the Cassini spacecraft that left Earth in 1997 and arrived at the planet Saturn in November 2004. This trip took seven years! But as you can see, after it was launched it traveled *away* from Saturn to Venus. Then its trajectory is an ellipse and visited Venus again and then back to Earth where it came from. After this quick visit to us it finally traveled outwards via Jupiter to finally Saturn. Now you may wonder: "why oh why did NASA choose this crazy path?" But of course there is a perfectly good reason for all this.

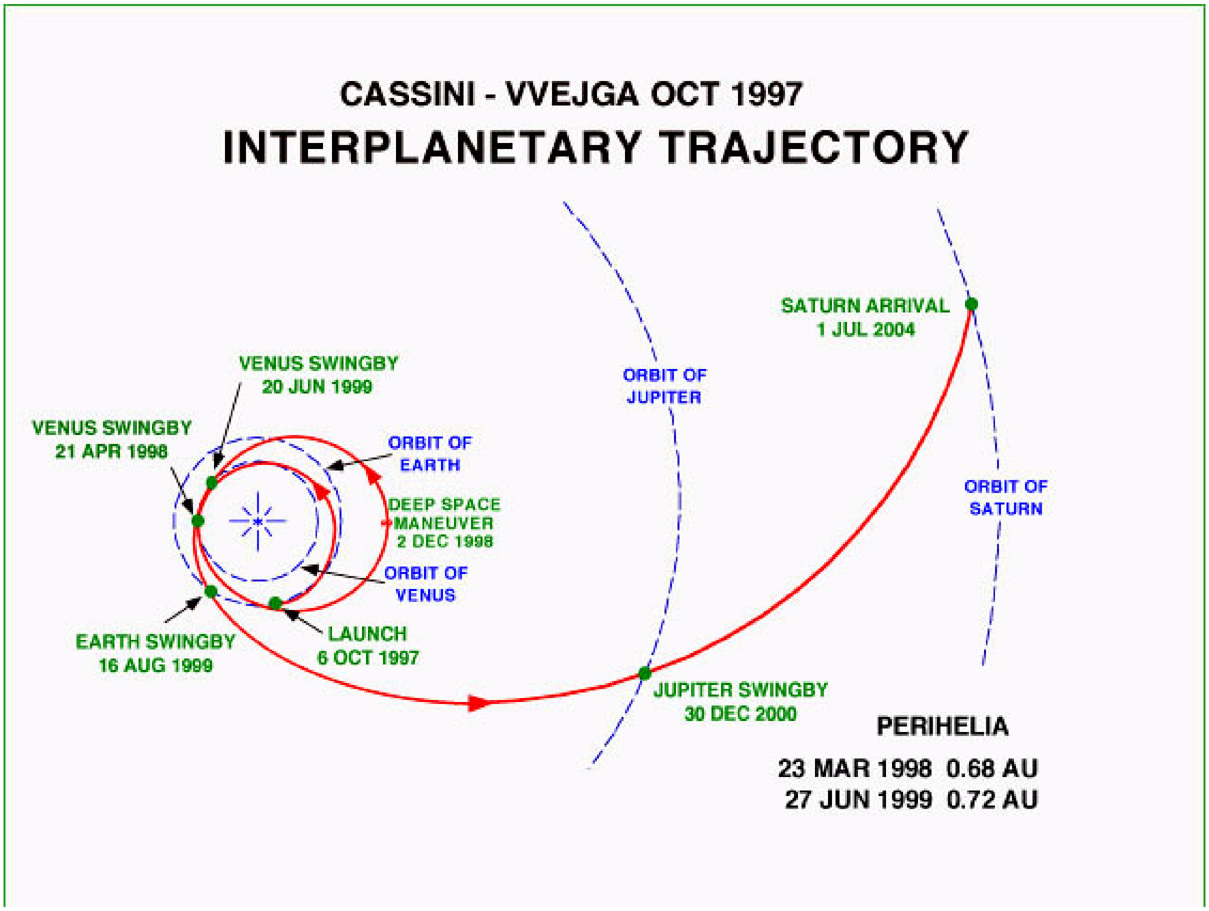


Figure 1

Cassini is a very large and heavy spacecraft; it's about as big as a bus! It would be incredibly difficult to give Cassini enough speed to fly to Saturn in one time. Let's take a closer look what exactly happens when a body (or a spacecraft) flies by a planet. In *figure 2*, the line represents the path of the spaceship and the circle is a planet.

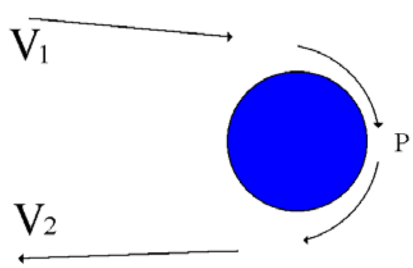


Figure 2

Our space craft arrives with a velocity of V_1, during the first part of the trajectory our craft will accelerate because in principle it is *falling* towards the planet. It will accelerate to point *P*, where it will reach its maximum speed, after which it will start to decelerate.

Problem 3:

a) How much will V_2 be?

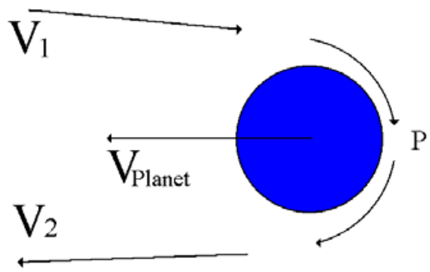


Figure 3

Of course our schematic above is too simplistic and we ignored many things, for example the planet itself will have a velocity as well. In *figure 3* we see the following representation; the spacecraft arrives as indicated, flies through point *P* and flies again away from the planet. The planet itself will have a velocity of V_{planet} and the velocity of the craft is again V_1 at arrival and V_2 after the passage. An observer on the planet will see our spacecraft arrive with a velocity of V_1+V_{planet} and leave again with a velocity of V_2-V_{planet}. Also for the observer on the planet the trajectory will be symmetric, in other words $V_1+V_{planet}= V_2-V_{planet}$ must hold true just like in *figure 1*. If we solve the above equation we get:

$$V_2 = V_1 + 2V_{planet}$$

Our spacecraft has gained a speed of $2V_{planet}$ without using any fuel!

Problem 4:

a) Calculate the orbital velocity of Venus around the Sun. Remember that the gravitational pull from the Sun and the Centrifugal force on Venus are exactly the same but opposite. Furthermore you can assume that the orbit of Venus is a perfect circle. More specifics you can find on the formula-sheet at the end of this paper.
b) If we assume that the Cassini spacecraft arrives at Venus with a velocity of 12 km/s, calculate the speed of Cassini after a passage with Venus.

Even if Cassini had to travel quite a ways in order to get this acceleration, the effect is clear. Even more so when you realize that with the chosen path of Cassini the total reduction in fuel was about 68,000 Kg fuel!

Problem 5:

a) What would happen if our satellite in Figure 3 would make the pass the other way around? When would this be useful?
b) If our satellite has again a starting velocity of 12 km/s what would happen?

You see that with smartly choosing a flight path you are able to reduce fuel costs significantly completely for free. The only ‘cost’ is that you have to be patient. One big problem is though that Venus, Earth, Jupiter, and Saturn all had to be in the *exact* right position. The two Voyager spacecraft that flew through the ‘whole’ Solar System also used this technique. The orbits that they followed were so special that they only occur once every 150 years. So you see: there are great benefits to gain but you need to be lucky.

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Sailing to Mars

Because of the many difficulties that traveling to another planet brings people often think or dream of different methods to do this. Most of these are pure science fiction and will probably never become reality. Others are technological ridiculous or just too expensive, or based on technologies that are pure fiction. For example think of the popular television series Star-Trek, where they beam people up and down without any effort. Of course this is a cool idea, but not really realistic. But there are some theories and ideas that are very promising and maybe possible in the near future. One of those ideas is a solar-sail. A sailboat moves because the wind blows in the sails. Solar-sails work very similarly, only we do not use wind (there is no wind in space), but we use literally the light of the Sun. This might sound strange because as you know a photon does not carry any weight and therefore cannot carry any momentum but this is not entirely true. A photon indeed has no mass, but most definitely has a momentum albeit a very small one. Assume for the following problems that our Sun radiates light of only one frequency: $\lambda = 5{,}75\text{X}10^2$ nm. Furthermore assume the radiated energy of the Sun to be $3.827\text{x}10^{26}$J, and that this energy is radiated spherically symmetrical. Our spacecraft weighs 10 metric tons and this includes the sail.

Problem 6:

a) Calculate the momentum of one single photon.
b) Calculate the number of photons that arrives by the Earth per m^2 per second
c) Calculate the momentum that the Sun delivers per m^2.
d) If we now use material that is 100% reflective and with a surface area of $1\text{x}10^6$ m^2, calculate the total momentum on this sail if the Sun shines at the sail at a right angle.
e) Calculate the acceleration of our spaceship.

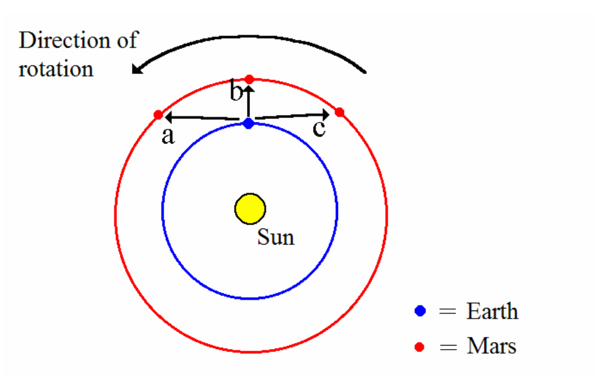

Figure 4

Maneuvering in space is not as a simple matter as it is on Earth. If you brake with your bike or car your speed will diminish, but you are still going in the direction before you started to brake. This is not the case in space. Take a look at figure 4. Here we depict the orbits of the Earth and the planet Mars. In principle there are three ways to travel from the Earth to Mars: path *a*, *b* or *c*. Do remember that the trip will take a while and that during that time the planets will continue to rotate. Imagine we take route *c*, in other words we are traveling *towards* Mars, but at the same time *against* the rotational direction of the two planets. If we accelerate from the Earth in that direction we *de*celerate from the Sun! That is a complex notion, but if you make a small sketch on paper you will see how this works.

Problem 6:

a) Explain why we will never arrive at Mars traveling path *c*.
b) Show that path *a* is the only correct path.

The same holds true when the Space Shuttle wants to make contact with the Space Station or a satellite for example. If you go slower you go *down* when you want to go up you need to accelerate. Maneuvering in space really goes against your gut-feeling! If we apply this knowledge to sailing through space we have two possibilities: **a** traveling *away* from the Sun (like to Mars), or **b** traveling *towards* the Sun (for example to Venus).

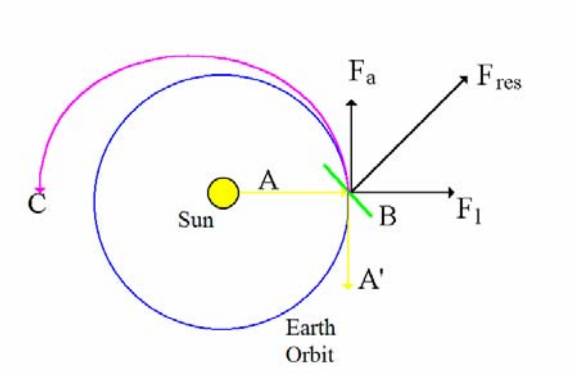

Figure 5

Let's examine option **a** more closely. We sketched the situation in figure 5. The spaceship (*B*) is in orbit around the Sun in Earth orbit and receives light from the Sun (*A*). Imagine the sail is under a 45^o degree angle, so the light is being reflected downwards towards *A'*. As a result of the incoming light there will be a force on the sail (F_{res}) which we can resolve into a vertical component (F_1) and relative to the Sun an accelerating force (F_a). Because of these forces our spaceship will change its orbit around the Sun, in this case a larger orbit. An orbit like this would bring us to for example Mars.

Problem 7:

a) Sketch in the figure below (Figure 6) what the position of the solar sail has to be in order to travel to Venus.
b) Sketch the forces just like in figure 5.

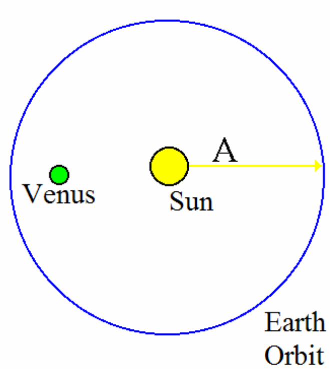

Figure 6

The formula that calculates the approximate force on the sail is:

$$F = ma = \frac{ALr}{2R^2 c\pi}$$

Where *L* is the light force of the Sun, A the surface area of the sale, r the reflecting factor[1], *R* is the distance from the sail to the Sun and lastly *c* is the speed of light. Realize that the mass *m* of the space ship is the ship *and* the sail together. Take for L = 3.83×10^{26} watts.

Problem 8:

a) Calculate how large the acceleration will be for a spaceship in orbit around the Earth of 1 ton (excluding sail), with a solar sail of 10 km x 10 km. The sail is aluminum and has a thickness of 1 micron.
b) How long will a trip to Mars take is we assume we have to travel 800 million km?

Apparently space travel will always ask a lot of patience. But still this is a large improvement because if we want to plan a manned mission to Mars, the craft will have to return as well. With conventional spaceships about 99% of the total weight of the craft at launch will have to be fuel! That is not only expensive but technically also very difficult to achieve. In addition it is quite dangerous as well. Imagine that indeed the future of space exploration will be done by sail 'ships' that float majestically from planet to planet. Wouldn't that be fantastic?

[1] If for example the reflecting material is aluminum, then about 88% of the light is reflected and 12% absorbed. In that case *r* will be 0.88